Standard deviation and percentile calculation

The following table shows the rating (0 being the worst and 10 the best) that a sample of 42 individuals assigned to the economic management of a government.

Lower Limit	Upper Limit	f_i
0	2	3
2	4	3
4	6	15
6	8	16
8	10	...

Calculate the sample standard deviation and determine the 25th percentile, explaining its significance in this case.

Solution

We calculate the missing frequency for the last class:

$$\text{Total frequencies} = 42 \quad \Rightarrow \quad f_5 = 42 - (3 + 3 + 15 + 16) = 5$$

Updating the table:

Lower Limit	Upper Limit	f_i
0	2	3
2	4	3
4	6	15
6	8	16
8	10	5

Calculate the midpoints x_i:

$$x_i = \frac{\text{Lower Limit} + \text{Upper Limit}}{2}$$

Updated table with calculations:

Class	Lower Limit	Upper Limit	f_i	x_i
1	0	2	3	$\frac{0+2}{2} = 1$
2	2	4	3	$\frac{2+4}{2} = 3$
3	4	6	15	$\frac{4+6}{2} = 5$
4	6	8	16	$\frac{6+8}{2} = 7$
5	8	10	5	$\frac{8+10}{2} = 9$

Calculate $f_i x_i$:

x_i	f_i	$f_i x_i$
1	3	$3 \times 1 = 3$
3	3	$3 \times 3 = 9$
5	15	$15 \times 5 = 75$
7	16	$16 \times 7 = 112$
9	5	$5 \times 9 = 45$
Total	**42**	**244**

Sample mean:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{244}{42} \approx 5.8095$$

Calculate $(x_i - \bar{x})$, $(x_i - \bar{x})^2$, and $f_i(x_i - \bar{x})^2$:

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	f_i	$f_i(x_i - \bar{x})^2$
1	-4.8095	23.1379	3	69.4138
3	-2.8095	7.8954	3	23.6861
5	-0.8095	0.6552	15	9.8276
7	1.1905	1.4173	16	22.6768
9	3.1905	10.1879	5	50.9395
			Total	**176.5438**

Sample variance:

$$s^2 = \frac{\sum f_i(x_i - \bar{x})^2}{n-1} = \frac{176.5438}{41} \approx 4.306$$

Sample standard deviation:

$$s = \sqrt{s^2} = \sqrt{4.306} \approx 2.075$$

Calculate the 25th percentile position:

$$P_{25} = \frac{25}{100} \times n = 0.25 \times 42 = 10.5$$

Cumulative frequencies:

Class	f_i	$CumulativeFrequency$
1	3	3
2	3	$3 + 3 = 6$
3	15	$6 + 15 = 21$
4	16	$21 + 16 = 37$
5	5	$37 + 5 = 42$

The 10.5th position is in class 3 (4 - 6), as $6 < 10.5 \leq 21$.
Formula for percentiles in grouped data:

$$P_k = L_i + \left(\frac{P - F_{i-1}}{f_i}\right) \times c$$

Where:

- $L_i = 4$: lower limit of the class containing the percentile.
- $F_{i-1} = 6$: cumulative frequency before the class.
- $f_i = 15$: frequency of the class containing the percentile.
- $c = 2$: class width.
- $P = 10.5$: percentile position.

Calculate:

$$\begin{aligned} P_{25} &= 4 + \left(\frac{10.5 - 6}{15}\right) \times 2 \\ &= 4 + \left(\frac{4.5}{15}\right) \times 2 \\ &= 4 + (0.3) \times 2 \\ &= 4 + 0.6 \\ &= 4.6 \end{aligned}$$

This means that 25% of the surveyed individuals rated the government's economic management with a score of 4.6 or lower.